Projectile Motion Notes – Projectile's Launched at an angle

From our notes before, we said that horizontal and vertical motions create projectile motion.

We also said that we must think about horizontal and vertical motions independent from one another.

This means we need to have formulas for both horizontal and vertical motions.

Here are the formulas:

Horizontal Motion	Vertical Motion
$v_x = \frac{d_x}{t}$	$d_y = V_{iy}t + \frac{1}{2} gt^2$
	$V_{fy} = V_{iy} + gt$
	$V_{fy}^2 = V_{iy}^2 + 2gd_y$

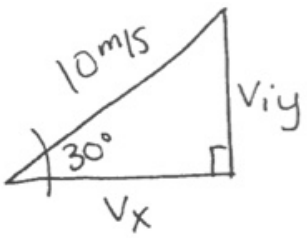
Example #1:

A soccer ball is kicked with an initial velocity of 10 m/s at an angle of 30° to the horizontal.

Picture:

Step 1: Resolve velocity into horizontal and vertical components.

Picture:



$V_x = v_i \cos\theta$

$V_{iy} = v_i \sin\theta$

$V_x = 10\,m/s \cos 30° = 8.66\,m/s$

$V_{iy} = 10\,m/s \sin 30° = 5\,m/s$

a.) Find the time of flight of the projectile:

In these problems... these objects have a final velocity (v_f) of 0 m/s at the **top of the object's path** (this is only ½ way though the projectile's path).

First: Use $v_{fy} = v_{iy} + gt$ -- to find ½ time.

$0\,m/s = 5\,m/s + (-9.8\,m/s^2)(t)$

subtract 5 m/s: $-5 = (-9.8\,m/s^2)t$

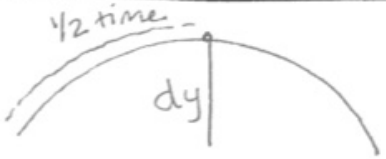
divide -9.8: $\frac{-5}{-9.8}$

.51 s = t = half time

Multiply ½ time by 2 to get full time:

.51 s × 2 = 1.02 s = full time

b.) Find the maximum height of the projectile: ← use $\frac{1}{2}$ time



Use $d_y = V_{iy}t + \frac{1}{2} gt^2$

$d_y = (5\ m/s)(.51s) + \frac{1}{2}(-9.8\ m/s^2)(.51s)^2$

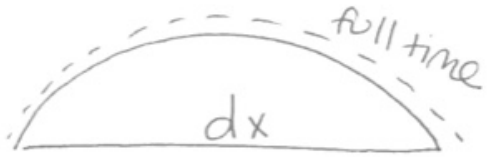
$d_y = 1.28m$

c.) What is the horizontal distance (range) the ball travels? ← use full time

Use $d_x = v_x t$

$d_x = (8.66\ m/s)(1.02s)$

$d_x = 8.83m$



2. A football is being kicked with a velocity of 15 m/s at a 24° angle to the horizontal.

Picture:

$V_{ix} = V_i \cos\theta = 15 \cos 24 = 13.7\ m/s$

$V_{iy} = V_i \sin\theta = 15 \sin 24 = 6.1\ m/s$

Find the full time the projectile was in the air:

Formula: $V_{fy} = V_{iy} + gt$

Plug in numbers: $0 = 6.1\ m/s + (-9.8\ m/s^2)(t)$

$\frac{-6.1}{-9.8} = \frac{-9.8t}{-9.8}$ (subtract 6.1 from both sides)

Answer: $\frac{1}{2}t = .622s$ full time $= 1.24s$

Find the maximum height the football traveled.

Formula: $d_y = V_{iy}t + \frac{1}{2}gt^2$

Plug in numbers: $d_y = (6.1\ m/s)(.622s) + \frac{1}{2}(-9.8\ m/s^2)(.622s)^2$

$d_y = 1.9\ m$

Answer:

What is the horizontal distance (range) the football travels?

Formula: $d_x = V_x t$

Plug in numbers: $d_x = (13.7\ m/s)(1.24s)$

Answer: $d_x = 16.99\ m$